# Pricing Renewable Identification Numbers (RINs) under Uncertainty

Mohamad Afkhami [1], Hamed Ghoddusi [2]

**Abstract**

We offer a novel modeling framework to determine the price dynamics of renewable identification numbers (RINs) -a *floor-and-trade* market-based mechanism for enforcing renewable energy standards. Our inter-temporal modeling approach is different than the usual practice which prices RINs in a static way. Using a continuous-time stochastic control formulation, we explicitly model the option value embedded in the RINs prices as an American spread option by taking into account the specific institutional constraints. We derive a closed-form solution of the RINs prices when underlying commodity prices are geometric Brownian motion (GBM). We also characterize the solution for setups with mean-reverting and jump specifications for the underlying prices, which need to be solved numerically. We propose a tight numerical approximation using duality methods. Among other results, we show that the price of RINs has a U-shape relationship with the volatility of ethanol and gasoline prices and a *negative* relationship with the correlation between the two price processes. We also show that once one of the underlying prices experiences high volatility, RIN prices converge to a fixed level irrespective of the volatility of the other process. The analytic framework can be used to model the dynamics of certificate prices in other markets with similar mechanisms (especially environmental service markets).

*Keywords:* Renewable Energy, Embedded Options, Floor and Trade Mechanism, Spread Option Pricing, Least Square Monte Carlo

## 1. Introduction

Biofuels have become an integral part of renewable energy portfolios in many countries around the world. In particular, the United States and EU have specific mandates for the minimum share of biofuels in the transportation (and in some cases electricity) sectors'

[1]Stevens Institute of Technology. Address: School of Business, Stevens Institute of Technology, 1 Castle Point on Hudson, Hoboken, NJ 07030, USA. Email: mafkhami@stevens.edu

[2]Stevens Institute of Technology. Address: School of Business, Stevens Institute of Technology, 1 Castle Point on Hudson, Hoboken, NJ 07030, USA. Email: hghoddus@stevens.edu, web:www.ghoddusi.com, Corresponding Author

energy inputs. One key question in this regard is how to enforce the minimum share of biofuels in practice. In this paper we focus on modeling the price dynamics of Renewable Identification Numbers (RINs), a market-based mandate enforcement mechanism.

We use a stochastic dynamic framework to better understand the value of a RINs certificate over time and solve the model under various assumptions. The renewable identification number (RIN) is a *floor-and-trade* mechanism (analogous to cap-and-trade mechanism of emission markets) to ensure that at the *aggregate level* , the required minimum quantity of biofuels has been blended with fossil fuels. The floor-and-trade mechanism allows firms to go beyond the minimum required mandate and produce excess certificates, which can either be sold to other firms to be exercised in the same period (the static view based on cost heterogeneity) or saved to be used in future periods (the dynamic inter-temporal view).

The conventional framework assumes the price of a RIN consists of a "core value"[3] and an option value. The literature tends to ignore the option component partly because of modeling difficulties. In the conventional framework the price of RINs will be zero as soon as the static cost of blending ethanol is lower than the price of gasoline. In this scenario, blenders will voluntarily blend ethanol and mandates will become non-binding. There will be no demand for RINs because it is optimal for all firms to do the physical blending. This, however, does not fit to empirical observations, in which RINs prices are always positive.

We show that the static formulation ignores the important *option value* component of RINs price. If the problem is properly modelled in a stochastic inter-temporal setup and the option value is accounted for, the price of RINs will never become zero, even if mandates are not statically binding (i.e., even when it is cheaper to blend than produce pure gasoline). We show that for a reasonable range of parameter values, the option value of RINs can be

[3]The core value captures the incentive of heterogeneous producers to blend different percentages of ethanol (some higher and some lower than mandated) in their fuel mix, and then trade and use the RINs within the same period.

considerable and should not be ignored. This is in line with historic RIN prices (which are always positive) and in sharp contrast with the intuition behind models that neglect the option value of RINs as compared to the core value.

We model the price of RIN under multiple scenarios for the dynamics of underlying prices. The first case, resulting in sharp analytic results, is when the price of gasoline and the biofuel follow a Geometric Brownian Motion (GBM) process. The second case is when the prices follow the exponential of mean-reverting Ornstein-Uhlenbeck processes. Under the GBM setup, the value of RINs can be obtained using a closed-form formula. This formula may also be used to approximate the value of RIN once the possibility of diffusion jump in the underlying prices is taken into account. The mean-reverting setup, does not yield to a closed-form formula for the value, but is considered more realistic when one models mean-reverting commodity prices. The biofuel we focus on in this paper is corn ethanol, and thus, the RIN of our interest is D6. This is because D6 RINs have by far the largest RFS2 volume mandate in the US. However, the framework can be easily modified and applied to all other types of RINs based on the production pathway.

Once gasoline and ethanol prices are modeled as GBM, it is not optimal for blenders to engage in early RIN *exercise*[4]. Under this assumption, the blenders always blend the required amount of ethanol and exercise RINs only in the last day of the compliance period. In the mean-reverting scenario however, the blender may find it optimal to exercise RINs at earlier periods. We calibrate both models to historic price data and among other results show a negative relation exists between RIN prices and the gasoline-ethanol correlation. We also observe a U-Shape relation between RIN prices and the volatility of the underlying processes. Numerical simulations reveal that when the volatility of one of the underlying processes is sufficiently high, RIN prices converge to the same level. This convergence holds

[4]See section 3 for the definition of exercising RINs

whether one considers the penalty of failing to meet the EPA mandate levels to be finite or infinite. The banking/borrowing level allowance has a linear effect on RIN prices. With respect to this variable, RIN prices are at their lowest once no banking/borrowing is allowed. The increase in allowance increases the probability that a RIN would be transferable to the next compliance year, and thus increases its value. We also show that the more expensive gasoline spot prices are relative to ethanol, the less RINs are expected to be worth.

Our paper adds a novel aspect to the extant literature on the economics of renewable energy. The majority of papers that analyzed the value of RINs have assumed its price to reflect the *static* difference between ethanol supply and demand price as the basis of their work, e.g., McPhail et al. (2011), Whistance and Thompson (2014), Pouliot and Babcock (2015), Lade and Lin (2015), and Markel et al. (2016). In a recent study, Korting and Just (2017) use a partial equilibrium model to show that the core value of a RIN in equilibrium reflects the marginal cost of employing one additional ethanol-equivalent unit of biofuel for the blender. This is in accordance with how price of permits in the cap and trade literature is considered to reflect the marginal cost of abatement (Seifert et al. (2008)).

We are not the first paper to consider a “dynamic” (i.e., inter-temporal) model for RINs (e.g., Lade et al. (2016) and Zhou and Babcock (2017)); however, to the best of our knowledge this paper is the first to consider a dynamic model under uncertainty using a continuous-time formulation. The dynamic setup used by earlier studies focuses on the deterministic growth of prices; whereas, the valuation in our paper is formulated as an optimal stopping problem. This setup allows for capturing the effect of the second moment and uncertainty around mean. A contrasting result is that in the previously used models, RINs will only be generated when its price is rising with the interest rate. While once the valuation is formed as an optimal stopping problem under uncertainty, RINs will still be generated even if there is no expectations for the price increase.

Our work is also closely related to papers on the price dynamics of traded emission certificates in markets such as European carbon emission certification (ETS) (Carmona and Hinz (2011), Hitzemann and Uhrig-Homburg (2018)). Besides contextual differences, our work differs from the existing literature on that topic in two key mathematical aspects. First, the underlying process for ETSs is typically a single stochastic process; whereas, the value of RINs is driven by the difference of two stochastic process. The "spread option" feature of RINs makes the problem substantially more complicated. Second, we solve the problem for both GBM and mean-reverting price processes and compare the impact of time-series dynamics of underlying processes to the price dynamics of certificates.

In this paper, we are focusing on the specific case of a trade-and-floor mechanism in the US biofuels market. Our modeling framework, with some modifications, can be extended to other markets in which a firm can choose between purchasing a traded certificate from the market or completing a mandated task. The floor-and-trade policies have been becoming more popular by the governments in areas where they seek to incentivize a minimum (floor) level production of a certain good. The Solar Renewable Energy Credits (SREC) program is another example of policy that aims to encourage the generation of electricity through renewable sources. Currently India, Australia, Sweden, Belgium, Italy, and the UK have SREC markets where certificates are traded. Within the US, California and New Jersey are among the number of states that have created similar markets. Other examples of policies in the US include the Wetland Mitigation Banking Program (to offset acres of wetlands converted to other uses, through restoration and creation of other wetlands) and Conservation Mitigation Banking Program (as part of the Endangered Species Act).

The rest of this paper is organized as follows. We provide a summary of relevant literature in Section 2. Section 3 explains the setup of the model for the price dynamics of RIN. Section 4 describes the numerical analysis of the developed framework for the RIN prices. We provide quantitative results as well as relevant discussion and implications in Section 5 . Finally, Section 6 concludes and proposes future research. We also provide some basic institutional details on RFS and RINs, and further surveys the related literature in Appendix A.

## 2. Literature Review

Previous studies have used a number of models to explain what the core value of RIN reflects. In addition to the studies mentioned in the introduction, Zhou and Babcock (2017) use the competitive storage model to estimate the impact of ethanol and fueling investment on corn prices.

In a well received study, under a deterministic continuous-time setting, Rubin (1996) investigates the banking dynamics of emission trading in a finite horizon. The paper argues that permit prices must be equal to the marginal cost of abatement and grow with riskless interest rate. McPhail (2010) applied this setup to the RINs market under RFS2 to find the optimum level of banking without putting any constraint on banking/borrowing. Thompson et al. (2009) is another paper to model RFS2 in a dynamic setting. They conduct a stochastic analysis to assess the shocks of corn and oil markets on ethanol price with and without the existence of RFS2. However, these dynamic models do not consider or model the option value associated with RINs. The discrete-time dynamic programming approach is also not well-suited for illustrating the sensitivity of RINs prices to the dynamics of the underlying prices as well as parameters such as volatility and correlation. Our continuous-time formulation enables us to derive the RINs value as an explicit function of volatility parameters.

Applying real option framework has recently gained more attention in renewable energy literature. Santibañez-Aguilar et al. (2016) analyze the optimal planning of bio-refineries with respect to risk in the supply chain. In a similar paper, McCarty and Sesmero (2015) use this framework to study gasoline prices that trigger entry for investing in a corn stover-based cellulosic biofuel plant. Bai et al. (2016) study the effects of farmland use regulation in the form of a cap-and-trade mechanism on balancing food and biofuel production. Boomsma et al. (2012) analyze investment timing and capacity choice for renewable energy projects under two different support schemes of feed-in tariffs and renewable energy certificate trading. Ritzenhofen et al. (2016) compare the effects of renewable portfolio standards, feed-in tariffs and market premia in terms of their contribution to achieving affordability, reliability, and sustainability of electricity supply. Ghoddusi (2017) examines the short profits of a bio-fuel plant by postulating its value as a strangle option.

A strand of literature compares the effects of carbon intensity standards and renewable fuel share mandates. With a Pigouvian tax approach, Holland et al. (2015) show that achieving the same level of carbon emission reduction is far less costly under a cap and trade program compared to RFS2. Their analysis is based on the result that under the current RFS, ethanol prices reflect an implicit subsidy, while gasoline is priced as if it were taxed, a result reported by a number of earlier works including Lapan and Moschini (2012), De Gorter and Just (2009), and Lade and Lin (2013). Holland (2009), on the other hand, shows that in the presence of market power, renewable share mandates are more efficient than cap and trade systems. Rajagopal et al. (2011) perform a multi-criteria comparison of RFS, LCFS, and carbon tax to analyze the effect of these policies on fuel price and emission reductions. They find that if fuel policy is applied regionally and the economy is open to trade, RFS increases the share of renewable to conventional fuels, and may decrease domestic fuel prices and carbon emissions.

Another dimension of the RFS literature explores the impact of policy uncertainty on the incentive for investment in new technologies. For example Lade et al. (2016) estimate the effect of 'policy shocks' imposed by the EPA by reducing RFS mandates on compliance costs, commodity markets, and the market value of publicly traded biofuel firms. They argue that these shocks reduced the incentive to invest in the technologies required to meet the future objectives of the RFS. Other studies in this dimension include Miao et al. (2012), and Clancy and Moschini (2017).

The option value embedded in RINs makes them also similar to another class of contracts known as swing options. Swing options are a family of contracts that allow the owner to receive variable amounts of a commodity within a time period at fixed prices. These contracts are especially popular in hydro-power production planning, where physical transfer of the underlying commodity is subject to volume restrictions. By providing flexibility in terms of delivery time and volume, swing options provide protection against price fluctuations. Numerous studies have examined the value of swing contracts based on different specifications and assumptions. Examples include Jaillet et al. (2004), Carmona and Touzi (2008), Fleten and Wallace (2009), Wahab et al. (2010).

## 3. Model Setup and Valuation

### *3.1. Background*

We focus on corn ethanol (i.e., D6 RINs). However, this framework can be applied to all other types of RINs. For the D6 RIN, the static price of RIN reflects the difference between the adjusted ethanol[5] and gasoline prices. The true price is the sum of the static price plus the option value associated with RINs. This option value comes from the fact that some excess RINs can be stored and exercised later. Exercising a RIN means that the producer decides to blend less ethanol than the mandated level and instead submit a paper RIN from the storage. This allows the producer to make *inter-temporal* optimization over blending rates. We assume that besides the time value of money (i.e., interest rates), no other cost is associated with storing paper RINs.

To obtain the required amount of RIN the representative blender has the option of either producing the required ethanol or obtaining the RINs directly from the market. Similar to other EPA regulations, during each year, the blender is also allowed to bank or borrow a

[5]Note that across the paper, the price of ethanol should also be interpreted as the adjusted price of ethanol to control for the difference between the energy content of ethanol and gasoline.

| Term | Definition | Remark |
|---|---|---|
| $\bar{Q}_i$ | Total final fuel production in year $i$ | Exogenous and fixed |
| $\psi$ | Mandated blend level | Exogenously set by the government ($\psi = 10\%$ for US ethanol). |
| $\bar{R}_i$ | Total RIN mandate in year $i$ | $\bar{R}_i = \bar{Q}_i\psi$ |
| $z_t$ | Number of RINs obtained in day $t$ | Optimally chosen by the blender |
| $R_t$ | Total RINs accumulated up to time $t$ | Bounded by total production, mandate level, and banking/borrowing regulations |
| $\chi$ | Maximum Ethanol that can be blended in a day | Exogenous and subject to physical constraints |
| $\xi$ | Borrow/banking constraint | Exogenously set by the government |
| $V$ | Price of a marginal RIN | Endogenously determined in the model |
| $T_1$ | Compliance date in the current year | $\bar{R}_i$ RINs need to be delivered. |
| $T_2$ | Compliance date in the next year | A fraction of current RINs can be transferred. |
| $P_E$ | Ethanol price | Stochastic process |
| $P_G$ | Gasoline price | Stochastic process |
| $r$ | Risk-free interest rate | Time-invariant, $r = 2.55\%$ |
| $\sigma_i$ | Volatility of gasoline and ethanol price processes | Time-invariant parameter of the model |
| $\rho$ | Correlation between ethanol and gasoline prices | Time-invariant parameter of the model |
| $\tilde{\alpha}_i$ | Mean-reversion rates of ethanol and gasoline processes | Time-invariant |
| $\exp(\tilde{\theta}_i)$ | Asymptotic mean reversion level of ethanol and gasoline processes | Time-invariant parameter of the model |

Table 1: Notations Used in the Model

certain percentage of its RINs. This provision can be explained as follows: absent banking/borrowing, the blender has to meet the mandate level in each banking period and once the compliance date passes, any excess RINs would be worthless. The introduction of banking/borrowing provisions allows the blender to minimize costs by obtaining more or less than the mandate level and carry forward/backward, based on their expectation of future. The industry as a whole would also become able to carry net deficit/surplus.

Table 1 introduces the notations used in the setup process.

*3.2. Key Assumptions*

We assume the industry consists of a continuum of homogeneous final gasoline producers (aka blenders) with no market power. Therefore, the market for RINs can be modelled through the optimization problem of a representative producer. Following the standard tradition of asset pricing, the representative producer is both seller and buyer of RINs. The equilibrium price of the *marginal* RIN in the market is determined by considering the marginal value of a RIN for the representative producer.

The producer is assumed to supply a total $\bar{Q}_i$ units of final fuel during each year[6] $i$. Government mandates requires the producer to include $\psi$ (volume) percent of ethanol in each unit of final fuel. Thus, the producer is obliged to deliver $\bar{R}_i = \bar{Q}_i \psi$ units of RINs by the end of that period.

If $z_u$ shows the number of RINs obtained in day $u$ ($0 \leq u < t$), then the total inventory of RINs is given by $R_t = \int_0^t z(u)du$, subject to $0 \leq z \leq \chi$. The latter constraint is enforced because blenders can blend only ethanol at a maximum proportion rate $\chi$ that depends on factors such as the type of biofuels, the maximum amount of ethanol that technically and legally can be blended in each product, and the production capacity. In practice many blenders can only produce a gasoline with maximum 15% ethanol, out of which 10% is the minimum mandate. Therefore, in reality the market can only produce 5% excess RINs in a day to be used later. In summary, the constraint ensures that the industry cannot generate an infinite amount of RINs in a single day.

*Benchmark: Static Pricing.* The price of ethanol, $P_E$, and gasoline, $P_G$, are random variables driven by Markovian stochastic processes. The static view of RIN value postulates that the value of RIN is given by:

[6] We assume $\bar{Q}_i$ is given for our problem. In reality the level of production is endogenous and a function of gasoline and ethanol prices. However, determining the optimal level of production is beyond the scope of this model. We fix the total quantity of the production in order to focus on the value of RINs.

$$V = \max[0, P_E - P_G] \quad (1)$$

In the static view, the blender always compares the opportunity cost of blending one unit of ethanol (and paying $P_E$) versus the cost of buying a unit of RINs from the market. In equilibrium, the marginal producer should be indifferent between buying RINs and blending ethanol. If $P_E < P_G$, then the price of RINs will be zero because all producers will voluntarily inject the cheap ethanol to the fuel mix.

*Dynamic Pricing.* The dynamic view suggests that the producer solves a stochastic optimal control problem over $z(u)$ - as the control variable - by considering the fact that at the end of the period $\bar{R}_i$ units of RINs must to submitted to authorities; however, the producer can blend more in some days (likely when ethanol is cheap) and then blend less in some other days (most likely when the realized price of ethanol is high); she can even borrow from future periods or begin by some RINs inventory inherited from previous periods. Note that under an unrealistic limiting case if the representative producer does not have sufficient flexibility in changing the blending rate (e.g. when she needs to blend exactly 10% everyday), the dynamic view disappears and the dynamic value converges to the and static values. However, as long as the producer has the luxury of changing the blending rate in a range, the option value of RIN becomes important.

*Marginal RIN.* We focus on the value of a marginal RIN which has been produced in the past and is traded in the market. Economic theory suggests that when there are multiple units of a good in a market, the equilibrium price is always determined by the value of the last (i.e. marginal) unit of that good. Even if legal and technical limitations over the range of blending (e.g. blender can not add 30% ethanol to the fuel mix or go negative on the ethanol share in the fuel) may force the representative producer to exercise the inventory of RINs over several periods, the marginal RIN can be freely exercised on any arbitrary day (as long as the current inventory of RINs is not larger than the total mandate.) Thus, we do not need to solve a complicated multi-unit exercise problem for the pricing purposes. Note that the marginal and the average prices of RINs will be different due to the inventory and limited flow capacity effects.

*Basic Valuation Equation.* Let $\mathbb{F} = \{\mathscr{F}_t\}_{0\leq t\leq T}$ denote the natural filtration generated by the price dynamics processes. The real world probability measure is represented by $\mathbb{P}$ and $\Omega$ is the set of all possible outcomes. Hence, $\{\Omega, \mathbb{F}, \mathbb{P}\}$ denotes the complete stochastic basis for the probability space for the random variables of interest. The value of the marginal RIN in the market over this probability space can be defined as in Definition 1.

**Definition 1.** *Let $R_t$ denote the total number of RINs obtained from the first day of the compliance period up to time $t$. The value of a RIN generated at time $t$ ($V_t$), under $\xi$% banking and borrowing constraint is given as:*

$$V_t(P_E, P_G, R_t) =$$
$$\begin{cases} (1-\xi)\sup_{\tau_1} e^{-r(\tau_1 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_1}[(P_E - P_G)^+] + \xi \sup_{\tau_2} e^{-r(\tau_2 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_2}[(P_E - P_G)^+] & R_t \leq \bar{R}_i \\ \frac{\bar{R}_i}{R_t}\left((1-\xi)\sup_{\tau_1} e^{-r(\tau_1 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_1}[(P_E - P_G)^+] + \xi \sup_{\tau_2} e^{-r(\tau_2 - t)}\mathbb{E}^{\mathbb{Q}}_{\tau_2}[(P_E - P_G)^+]\right) & R_t > \bar{R}_i \end{cases} \tag{2}$$

*where $\mathbb{Q}$ is risk-neutral measure, and $\xi$ is the borrowing/banking cap. The notation $\mathbb{E}_t[\mathcal{A}]$ represents the expectation of $\mathcal{A}$ conditional on the filtration $\mathscr{F}_t$. $\tau_i = T_i - t$, $0 \leq t \leq \tau_1 \leq T_1 \leq \tau_2 \leq T_2$, and $T_1$ and $T_2$ represent the compliance date in the current and consecutive year, respectively.*

We observe that the RIN value is a function of three state variables: current gasoline prices ($P_G$), current ethanol price ($P_E$), and the current inventory of accumulated RINs ($R_t$). A RIN is **exercised** at the optimal stopping time $\tau_i$ if the blender decides to use an equivalent volume of gasoline instead of blending ethanol at that time.

Definition 1 suggests that the value of RIN is composed of two American exchange options (i.e., a spread option with strike price equal to zero), with the same underlying processes and different time to maturity[7]. By the end of the compliance period, the blender has to meet the obligation of that year. This is done while $t \leq T$ by either blending physical ethanol at cost $P_E(t)$ or by exercising some of existing RINs. The flexibility to choose one of these two methods indeed creates the option value for the producer.

The decision between the two choices (blending or using RINs) is continuously made at each time $t$. Obtaining RINs gives its holder the *option* to choose when to exercise them later. This in essence is the fundamental *option value* associated with RINs.

*Effect of RIN's Inventory.* An important feature of the valuation is the effect of meeting the compliance level (i.e., obtaining the required number of RINs, $\bar{R}_i$) *before* the compliance date. As long as the economy has not yet produced enough RINs to meet the mandate level, RINs are valued similar to the American option as described. Once the required RINs for that period have been generated and accumulated, not all RINs would have a chance to be exercised. When more RINs are produced than the required mandate, inevitably some would not be submitted to the EPA. Those RINs not submitted to EPA and not banked for the next year, would become worthless. Therefore, after the $\bar{R}_i$ threshold the value of a marginal RIN in the remaining days would be multiplied by its chance of being submitted to EPA (i.e. $\frac{\bar{R}_i}{R_t}$). Once the threshold, $\bar{R}_i$, is met, each RIN has only a probability (less than one) of being used. Since RINs of the same vintage are assumed identical in nature, this probability is simply defined as $\frac{\bar{R}_i}{R_t}$. It can also be directly inferred that whether or not the required level of compliance ($\bar{R}_i$) has been met, the value of a RIN of last year's vintage, i.e., that expires in the current year mandate, lacks the second term in Definition 1 in its value.

[7] To be more precise, RINs are in fact Bermudan options as they can only be *exercised* in a discrete set of time. We address this distinction when the underlying prices are assumed to be mean-reverting.

### 3.3. Price Processes

In order to solve the valuation model, one needs to introduce specific functional forms for the stochastic processes governing ethanol and gasoline prices.

One key difference between financial securities (e.g., stocks) and commodity prices is that commodity prices tend to have slow reversion and some long-run equilibrium levels despite their random evolution (Clewlow and Strickland (2000), Carmona and Durrleman (2003), and Eydeland and Wolyniec (2003)). The mean-reversion property is triggered by equilibrating forces such as new and alternative sources of supply, entry and exist of suppliers, efficiency measures, switching to other energy sources, and the demand cap. Schwartz (1997), and Gibson and Schwartz (1990) are among the first studies to utilize Ornstein-Uhlenbeck process to describe the price behavior of commodities. On the other hand, the efficient market hypothesis (EMH) postulate that the price of traded commodities should be close to random-walk. The choice of proper stochastic process (unit-root versus mean-reverting) has been an ongoing debate in the commodity finance literature.

In order to provide a comprehensive treatment of the matter, we solve the RINs price dynamics for both unit-root (GBM) and mean-reverting processes and compare the impact of this choice on the price dynamics. To avoid negative values, the price dynamics on the mean-reverting case are defined to follow the exponential of Ornstein-Uhlenbeck processes.

#### 3.3.1. Geometric Brownian Motion Processes

In the first setup, prices of ethanol ($P_E$) and gasoline ($P_G$) are assumed to follow geometric Brownian motion (GBM) processes under the risk-neutral measure $\mathbb{Q}$ following:

$$\frac{dP_i}{P_i} = \mu_i dt + \sigma_i dB_i(t) \tag{3}$$

where $dB_i(t)$ are Brownian shocks, $mu_i$ and $\sigma_i$ ($i \in \{E, G\},$) are time-invariant drift and volatility parameters and $\mathbb{E}\{dB_E(t)dB_G(t)\} = \rho dt$ is the correlation between the two processes is through the driving Brownian motions. Note that we assume the convenience yield to be negligible.[8] It has been proven (e.g., Broadie and Detemple (1997) and Bjerksund and Stensland (1993)) that in this setting, as there is no dividend, the problem of valuing an American Exchange option can be reduced to that of a European Exchange option. i.e., in our terminology, at all times before final compliance date, the blender always decides to produce ethanol rather than exercising the RIN. If at the end of the compliance period ethanol is cheaper than gasoline, the blender meets the mandate by producing ethanol, thus wasting the premium paid (the RIN price). However, if gasoline ends up being the more expensive fuel, the blender receives the price difference by exercising the option (i.e., submitting the RINs rather than physical blending). Thus, to anaylze the value of a RIN, one can first derive the value of each term separately and describe the total value as the sum of two terms. Margrabe (1978) proves a closed form price solution exists for this setting.

**Proposition 1.** *If the underlying processes are given by GBMs with correlation $\rho$, the price of RIN at time $t$ is given by:*

$$V_t = \begin{cases} (1-\xi)\big(P_E(t)\Phi(d_{1+}) - P_G(t)\Phi(d_{1-})\big) + \xi\big(P_E(t)\Phi(d_{2+}) - P_G(t)\Phi(d_{2-})\big) & R_t \leq \bar{R}_i \\ \frac{\bar{R}_i}{R_t}\left((1-\xi)\big(P_E(t)\Phi(d_{1+}) - P_G(t)\Phi(d_{1-})\big) + \xi\big(P_E(t)\Phi(d_{2+}) - P_G(t)\Phi(d_{2-})\big)\right) & R_t > \bar{R}_i \end{cases} \tag{4}$$

*where*

$$d_{1\pm} = \frac{\ln(P_E(t)/P_G(t))}{\sigma\sqrt{T_1 - t}} \pm \frac{1}{2}\sigma\sqrt{T_1 - t} \quad d_{2\pm} = \frac{\ln(P_E(t)/P_G(t))}{\sigma\sqrt{T_2 - t}} \pm \frac{1}{2}\sigma\sqrt{T_2 - t} \tag{5}$$

[8] the effect of convenience yield is later discussed in section 3.3.2.

*and:*

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du \quad \sigma = \sqrt{\sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G} \tag{6}$$

*with $T_1$ and $T_2$ being the last day in the current and consecutive year, respectively.*

*proof:.* See Appendix B.

It is interesting to note that the solution is independent of the risk free interest rate, $r$. This is because in a risk-neutral world (and absent a significant convenience yield), prices of ethanol and gasoline increase at the same discount rate, offsetting each other in the computation appearing the in the definition of RIN.

*3.3.2. Geometric Mean-Reverting Processes*

Under this setup, the dynamics of the underlying prices $(P_E, P_G)$ are assumed to be given by the exponential of two Ornstein-Uhlenbeck processes. We assume the risk-neutral dynamics of the prices are given by stochastic differential equations of the form:

$$\begin{aligned} P_i(t) &= \exp\left(\theta_i + X_i(t)\right) \\ dX_i(t) &= -\alpha_i X_i(t)dt + \sigma_i dB_i(t) \end{aligned} \tag{7}$$

where $i \in \{E, G\}$, $\alpha_i$s are the constant mean reversion coefficients. $B_E(t)$ and $B_G(t)$ are standard Brownian motions with correlation $\rho$ under the risk neutral measure $\mathbb{Q}$. $\exp(\theta_i)$ is known as the asymptotic mean reversion level and $\sigma_i$ is the volatility. Appendix C shows how the risk-neutral dynamic is derived from real world measure $\mathbb{P}$.

Unlike the case of GBM, one cannot ignore the possibility of early exercise under GMR assumption. In fact, with the mean-reversion property acting similar to a dividend yield or a convenience yield, the early exercise premium is expected to be positive. Intuitively, this can be demonstrated by considering instances where one of the prices reaches a very high deviation from its long-run mean. The mean-reverting property will dictate the process to "come back" to a lower level, which then makes the early exercise the optimal decision.

The American option valuation can thus be solved by framing the valuation as a free boundary problem and solving for the resulting partial differential equation (PDE). Setting $x = \ln P_E - \theta_E$ and $y = \ln P_G - \theta_G$ this value function, $V(t, x, y)$, must satisfy the following PDE:

$$\frac{\partial V}{\partial t} + \alpha_E(\theta_E - x)\frac{\partial V}{\partial x} + \alpha_G(\theta_G - y)\frac{\partial V}{\partial y} + \frac{1}{2}\left(\sigma_E^2 \frac{\partial^2 V}{\partial x^2} + 2\rho\sigma_E\sigma_G \frac{\partial^2 V}{\partial x \partial y} + \sigma_G^2 \frac{\partial^2 V}{\partial y^2}\right) - rV = 0 \quad (8)$$

So far we have assumed that firms must comply with the mandates regardless of underlying prices. However, in reality, the EPA enforces the mandates by penalizing firms that fail to meet the mandate levels. The effect of this penalty can be captured in the boundary conditions of the PDE.

Assume the blender faces a penalty of $\Pi_i$ for each unit of shortage in RINs. Consequently, once the price of ethanol becomes much higher than gasoline, an upper bound is imposed on the value of RIN: no blender will be willing to pay a price for the RIN higher than the penalty[9].

The PDE is governed by the following boundary conditions:

$$\begin{aligned} &\lim_{x\to\infty} V(t,x,y) = \Pi_i \quad \lim_{x\to 0} V(t,x,y) = 0, \\ &\lim_{y\to\infty} V(t,x,y) = 0 \quad \lim_{y\to 0} V(t,x,y) = P_E(t), \end{aligned} \quad (9)$$

satisfying the value matching

$$V(t, x^*(t, y^*), y^*) = e^{\theta_E + x^*(t,y^*)} - e^{\theta_G + y*} \quad (10)$$

[9]In this situation the firm is better off paying the penalty, rather than obtaining RINs at very high prices.

and the smooth pasting conditions:

$$\frac{\partial V(t, x^*(t, y^*), y^*)}{\partial x} = e^{\theta_E + x^*(t,y^*)} \quad \frac{\partial V(t, x^*(t, y^*), y^*)}{\partial y} = -e^{\theta_G + y*} \tag{11}$$

where $(x^*(t, y^*), y^*)$ is the trigger curve in the (x,y) plane as function of time.

## 4. Numerical Solution Approach

To this date, no analytical solution for pricing American options with mean-reverting underlying process(es) has been developed. Therefore, to value RINs that satisfy the PDE in Equation 8, one needs to utilize numerical techniques such as implicit, explicit or Crank-Nicolson finite difference methods, tree approximations, or Fast Fourier Transformations (e.g., Jaimungal and Surkov (2011), Jaimungal et al. (2013)).

We, instead, primarily take advantage of the least square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz (2001). The choice of this solution approach is driven by the fact that according to the Central Limit Theorem, the standard error of the estimate converges to zero at the order of $O(\frac{1}{\sqrt{N}})$, where $N$ is the number of simulated paths. The advantage that convergence only depends on the number of paths (and is independent of dimension) is particularly handy in our setup. In addition, unlike lattice-based methods, the computational effort required for the LSMC method increases only linearly as the number of stochastic factors increase.

We assume the choice between blending or using RINs for a blender is a decision made on a discrete time (e.g. on daily, weekly or monthly) basis. Consequently, pricing RINs can be treated similar to a Bermudan option with finite exercise dates. This is consistent with the LSMC method for pricing American options in which the option is approximated in the form of a Bermudan option with a set of exercise opportunity dates with the time between them being small. The LSMC provides a *lower bound* for the value of the RIN.

Using the primal-dual representation, one can also calculate an *upper bound* for the price through a minimization problem over a class of (super) martingales (established by Andersen and Broadie (2004), and Haugh and Kogan (2004)).

The logic behind LSMC is similar to the general simulation approach to pricing American options: at each exercise opportunity, the value of immediate exercise is compared to the expected payoff from continuation, i.e., holding on to the option. This set of decisions constructs the value of the option. LSMC combines least-square linear regression and backward induction to approximate the value of holding the option. After generating a set of paths for the underlying price processes, by using backward induction, the continuation value at each exercise date is estimated by regressing the discounted subsequent realized cash flows from continuation on a set of basis functions. The regression is only fitted on paths where the option is *in-the-money*. Finally, price of the option is the discounted cash flows.

### *4.1. Application of LSMC to RINs*

Setting the time of expiry at $T = t_n$, the value of RIN can be priced recursively on the exercise dates $m \in \{1, 2, ..., n\}$ as follows:

$$v_{t_m}(x, y) = \begin{cases} (e^{\theta_E + x} - e^{\theta_G + y})^+, & m = n \\ \max\left\{(e^{\theta_E + x} - e^{\theta_G + y})^+, C_{t_m}\right\}, & m \in \{1, 2, ..., n-1\} \end{cases} \tag{12}$$

where $C_{t_m} = e^{-r\Delta t_m}\mathbb{E}[v_{t_{m+1}}(X_{t_{m+1}}, Y_{t_{m+1}})|X_{t_m} = x, Y_{t_m} = y]$ is the value of holding the RIN and not exercising it at time $t_m$, which can be calculated as:

$$C_{t_m} = \sum_{r=1}^{d} \beta_{mr}\psi_r(x, y) \tag{13}$$

for some basis functions $\psi_r$ and coefficients $\beta_{mr}$.

We choose the basis functions based on the algorithm's ability to replicate the results of Haug (2007) which uses 3D binomial trees, and Jaimungal et al. (2013) which uses the Fourier Transform method. This leads to eight basis functions:

$$\begin{aligned} &\psi_1 = x, \quad \psi_2 = y, \quad \psi_3 = xy, \quad \psi_4 = x^2, \\ &\psi_5 = y^2, \quad \psi_6 = x^2 y, \quad \psi_7 = xy^2, \quad \psi_8 = x^2 y^2 \end{aligned} \tag{14}$$

Finally, to reduce the variance of simulations, we use the exact price of a European exchange option as the control variate. Li et al. (2008) provide a closed-form expression of the European exchange option price when the underlying processes are mean-reverting.

**Proposition 2.** *For $d$ basis functions, at each of the $m-1$ early exercise dates after time $t$, let $LSMC(x, y, d, m)$ denote the discounted cash flows resulting from following the comparison rule of equation 12 when the immediate exercise value is greater than or equal to $C_{t_m}$ defined in equation 13 for $d$ basis functions. Then the lower bound value of RIN can be approximated as:*

$$V_t = (1-\xi) \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} LSMC(x, y, d, m_1) + \xi\big(\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} LSMC(x, y, d, m_2)\big) \tag{15}$$

*for $R_t \leq \bar{R}_i$ and $\frac{\bar{R}_i}{R_t} V_t$ for $R_t > \bar{R}_i$. In the equation above, $N$ represents the number of generated paths that are in-the-money. Further, $m_1$ and $m_2$ represent the number of early exercise dates remaining in the current and next consecutive year.*

*proof:.* See Appendix D.

The upper bound of RIN price can be computed with the dual representation of Definition 1. Andersen and Broadie (2004) provide a detailed description on deriving the upper bound value of options. Here, we use those results to briefly describe the application process in this setup. Let $H$ represent the space of adapted martingales $\pi$ for which $\sup_\tau |\pi_t| < \infty$ and

$\pi_0 = 0$. For any martingale at time 0, the following relation holds:

$$v_0 = \sup_{\tau} \mathbb{E}_0^{\mathbb{Q}}[\frac{h_\tau}{e^{r\tau}} + \pi_\tau - \pi_\tau] = \pi_0 + \sup_{\tau} \mathbb{E}_0^{\mathbb{Q}}[\frac{h_\tau}{e^{r\tau}} - \pi_\tau] \leq \pi_0 + \mathbb{E}_0^{\mathbb{Q}}[\max_t \frac{h_t}{e^{rt}} - \pi_t]$$

where $h_t = \max{(P_E(t) - P_G(t))^+}$ and the last inequality holds because for any $t$ in the set of stopping times:

$$\mathbb{E}_0^{\mathbb{Q}}[e^{-rt}h_t - \pi_t] \leq \mathbb{E}_0^{\mathbb{Q}}[\max_{\tau} \left(e^{-r\tau}h_\tau - \pi_\tau\right)]$$

Using the fact that $\pi$ was arbitrary (meaning, the inequality should hold after taking infimum) an upper bound for the ethanol and gasoline spread is calculated as

$$\inf_{\pi \in H} \left(\pi_0 + \mathbb{E}^{\mathbb{Q}}[\max_t e^{-rt}((P_E(t) - P_G(t))^+ - \pi_t]\right) \tag{16}$$

This defines the dual problem. To achieve a "tight" upper bound, $\pi_t$ is defined as $\pi_0 = L_0$, $\pi_1 = e^{-rt}L_1$, and for $2 \leq m \leq n$ with $n$ being the number of exercise times

$$\pi_m = \pi_{m-1} + e^{-rt}L_m - e^{-r(t-1)}L_{m-1} - l_{t_m-1}\mathbb{E}[e^{-rt}L_m - e^{-r(t-1)}L_{m-1}] \tag{17}$$

with $l_{t_m-1}$ indicating the decision at $t_m - 1$ ($= 0$ for continuation and $= 1$ for exercise), and $L_t$ is the value of an option that is newly issued at time $t$. Then the upper bound is given by:

$$V^{up} = L_0 + \mathbb{E}^{\mathbb{Q}}[\max_{\tau} e^{-r\tau}((P_E - P_G)^+ - \pi_t] \tag{18}$$

The above valuation of the spread can then be used to estimate each of the terms in Definition 1 to calculate the upper bound value of RINs individually.

*4.1.1. Exercise boundary*

To analyze the exercise boundary or the trigger curve with respect to time to maturity it should first be noted that in the case of an American call option with dividend, the limiting exercise level does not always approach the strike price. Under GBM assumptions, achieving a limiting trigger level equal to strike requires large dividends. In the same fashion, in our setup the limiting trigger curve is not simply $P_E^* = P_G^*$. This may be explained by the argument that the mean-reversion property acts similar to a dividend yield, through a very different role. Jaimungal et al. (2013) prove that under mean-reversion assumptions, as time to expiry approaches, the limiting boundary of the trigger region follows a curve in the $(P_E, P_G)$ plane satisfying the constraint:

$$\frac{P_E^*}{P_G^*} = \max\left(1, \frac{\alpha_G(\ln(P_G - \theta_G) + (r - \frac{1}{2}\sigma_G^2)}{\alpha_E(\ln(P_E - \theta_E) + (r - \frac{1}{2}\sigma_E^2)}\right) \tag{19}$$

Recall that in the case of GBM with dividend, the limiting trigger of an American option approaches $P_E^* = P_G^*$ as the dividend yield increases. Using this fact, the intuition behind the trigger curve can be better illustrated. The increase in the difference between prices prices and long-term equilibrium levels (which acts similar to a dividend yield) brings the trigger curve closer to $P_E^* = P_G^*$. In addition, at lower volatilities -when the optionality of RIN reduces- curve becomes closer to $P_E^* = P_G^*$.

## 5. Quantitative Results and Discussion

To conclude the analytical exercise, we present quantitative (numerical) results of the model using some real-world parameters and also discuss some additional features of the model.

### *5.1. Numerical Results*

As discussed earlier, a set of arguments exists in favor of both the GBM and GMR as the correct specification of price processes. Before studying the quantitative results, we examine the unit-root behavior of the two price series to see whether one of the regimes is more appropriate in explaining the regimes based on historical data. We conduct the Augmented

Table 2: Unit-root test results

| variable | ADF | PP |
|---|---|---|
| Gasoline | -2.52 (0.36) | -8.46 (0.64) |
| Ethanol | -2.82 (0.23) | -16.01 (0.22) |

The numbers in the table are $t$-statistic values and $p$-values are reported in parenthesis. Estimates are based on weekly gasoline and ethanol spot prices between Jan 1st 2010 and Dec 1st 2017.

Dickey-Fuller (ADF) and Phillips-Perron (PP) unit-root tests for ethanol and gasoline prices. Results reported in Table 2 suggest that one cannot reject the null hypothesis of unit-root behavior for any of the series (at least the sub-sample of 2010-2017).

Moreover, as pointed out by Ghoddusi (2017) and further illustrated and examined by Afkhami et al. (2017), due to the weak power of the ADF tests and also due to regime changing behavior of ethanol and gasoline prices, one can not strongly reject one model in favor of the other for a longer period. Therefore, we provide quantitative results using both models. The comparison of results for two setups reveal additional insights too.

The rest of this section represents the results of implementing the models and numerical methods. The GBM and GMR models are calibrated to spot price data of gasoline and ethanol. Weekly spot prices between Jan 1st 2010 and Dec 1st 2017 are obtained from the EIA website. Prices are then fitted to the models by the method of maximum pseudo-likelihood of one dimensional stochastic differential equation. Parameter estimation results are reported in Table 3.[10] We use the 5-year treasury yield curve rate as the riskless interest rate.

#### *5.1.1. Effect of Volatility*

Figure 1 presents the impact of volatility on RIN prices. In particular, the four graphs show the relation between volatility of one of the underlying processes with the RIN value for various levels of volatility of the other underlying.

[10] The calibration is done using the `Sim.DiffProc` package provided by Guidoum and Boukhetala (2017).

Table 3: Parameter Estimates

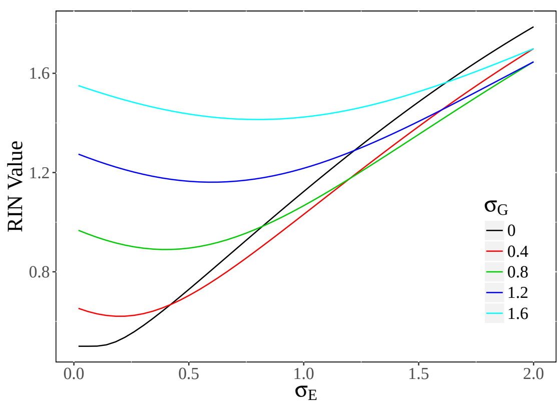
| Model | Parameter | Estimate |
|---|---|---|
| GBM | r | 2.55% |
| | $\mu_E$ | 0.00 |
| | $\mu_G$ | 0.00 |
| | $\sigma_E$ | 0.045 |
| | $\sigma_G$ | 0.035 |
| | $\rho$ | 0.85 |
| GMR | r | 2.55% |
| | $\theta_E$ | 1.05 |
| | $\theta_G$ | 0.81 |
| | $\alpha_E$ | 0.12 |
| | $\alpha_G$ | 0.10 |
| | $\sigma_E$ | 0.046 |
| | $\sigma_G$ | 0.037 |

Parameters used in numerical simulations. Coefficients are estimated based on weekly gasoline and ethanol spot prices between Jan 1$^{st}$ 2010 and Dec 1$^{st}$ 2017.

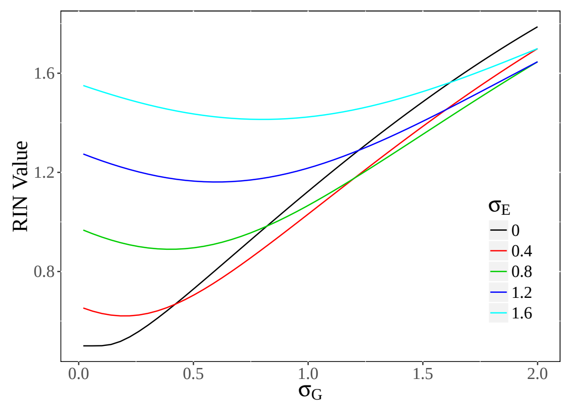
The sensitivity of RIN value to volatility provides a series of intriguing observations. First, as can be inferred from the analytical formula too, under the GBM assumption, the volatility of ethanol and gasoline both have the same effect on the RIN price. Second, a weak U-shape relation exists between volatility and price under both scenarios and for both processes. The third noticeable result is that under GBM, the price of RIN converges to the same common value once the volatility of ethanol or gasoline price increases. What makes this observation noteworthy is the fact that the convergence holds even if the penalty assumption is relaxed, and firms are expected to comply regardless. This means that even if the first boundary condition in equation 9 is changed to $\lim_{x\to\infty} V(t,x,y) = \infty$, RIN values converge. While the first mentioned feature can be directly inferred from the closed-form valuation, the two other features may be discussed in more detail.

**Proposition 3.** *Under GBM assumptions, as the volatility of one of the underlying prices increases, RIN value converges to $P_E$ when $R_t \leq \bar{R}_i$ and $\frac{\bar{R}_i}{R_t}P_E$ when $R_t > \bar{R}_i$.*

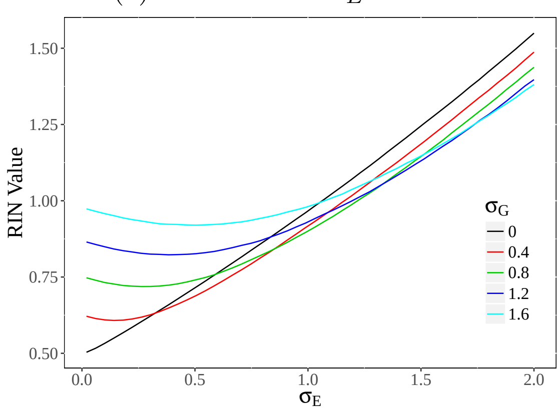
*proof.* : See Appendix E.



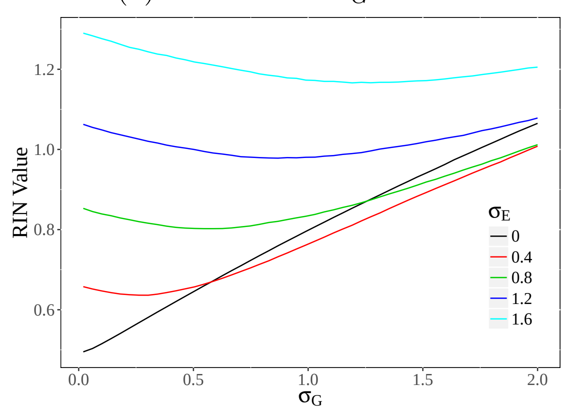

(a) The effect of $\sigma_E$ for GBM




(b) The effect of $\sigma_G$ for GBM




(c) The effect of $\sigma_E$ for GMR




(d) The effect of $\sigma_G$ for GMR

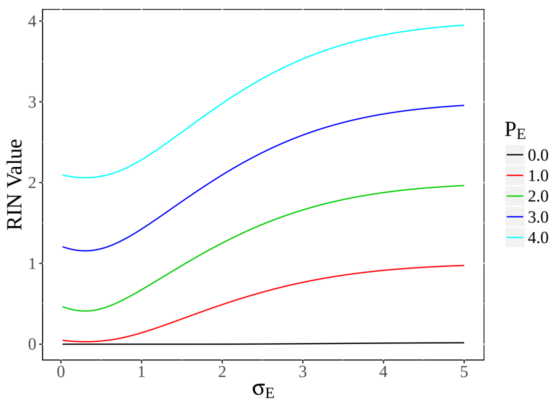
Figure 1: The effect of volatility on RIN value under both scenarios. The initial price values are set as $P_E = 2.5$, $P_G = 2.0$. Except volatility other parameters are held constant as reported in Table 3. We observe that when the volatility of one of the processes is sufficiently large, all RIN values converge to a common level.

To see why is the relation between volatility and RIN value negative at first and positive after a certain threshold consider the following thought experiment. Let us first focus on the negative part of the relationship. Consider a case with zero volatility of the ethanol and a positive volatility for gasoline. Also assume that the initial price level of ethanol in the case of GBM and its long term level (in the case of GMR) is higher than that of gasoline. In this setup, the variance of ethanol price is zero but there is a certain probability that the realized gasoline price would be greater than that of ethanol. Keep all parameters constant and only increase the volatility of ethanol to a small positive value. Here, the value of RIN also decreases slightly. The reason is because of the increased variance of ethanol, the probability of price difference ($P_E - P_G$) being negative (and thus the RIN being worthless) increases. But this negative relation holds only up to a certain threshold of ethanol volatility. After this level, even though the instances where the RIN value becomes zero increases, the probability of ethanol price achieving extremely high values also increases. In more details, in instances where the ethanol price is lower than gasoline price, it is not important how large the magnitude of this difference is. The RIN's value becomes zero regardless. With vary large realizations of ethanol price, however, the magnitude of difference has an impact on RIN value. The same logic explains why at very extreme levels of volatility for one underlying, the RIN value tends to converge for various levels of volatility of the other underlying: the realizations of one process become the dominant determinant of the expected pay-offs the in-the-money region.

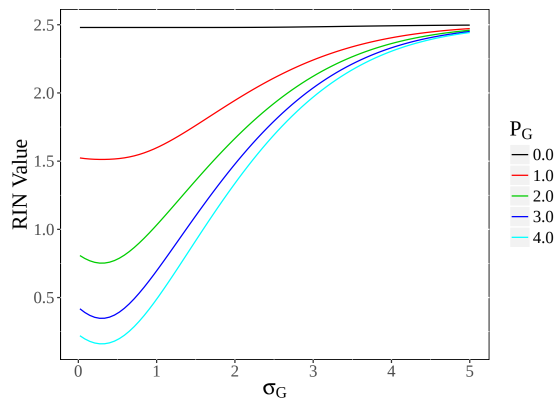
*5.1.2. Interaction of Volatility and Price Levels*

The effect of volatility for various initial prices (under GBM) and long-term equilibrium price levels (under GMR) is further studied in Figure 2. Again a U-shape relation is observed between volatility and RIN value. Figure 2-$a$ shows as the chance of $P_E$ being above $P_G$ increases (either due to increased volatility or a higher initial value), the value of RIN increases. The size of this increase has a positive relation with the price of ethanol. (or to be more exact, the difference between the price of ethanol and gasoline.) Figure 2-*b* clearly shows how the price of RIN converges the same value once the volatility of gasoline increases to very high levels. To see the reason, assume the initial price is 2.5 for ethanol and 1 for gasoline. If the volatility of ethanol is a positive number and volatility of gasoline is zero, discounted ethanol prices have the same probability to be above 2.5 by the compliance date. The expected RIN price would therefore remain very close to the static difference. Once the positive volatility of ethanol and the initial prices are held constant and one increases the volatility of gasoline, the probability of the gasoline price being above the ethanol price increases; an effect that decreases the value of RIN. This is where we are observing the downward slope of the U-shape graph. However, once the volatility of gasoline passes a certain threshold, the relation changes to positive. The intuition is again similar to that of Figure 1. Above this threshold of volatility, though it is possible to have very high and very low levels of gasoline values, the only values that effect the price of RIN are those that still keep the price difference positive. In other words, for extreme levels of volatility, gasoline prices have a non-negligible probability of achieving levels (much) higher than ethanol, but at the same time they also have a greater chance to be at very low levels. While for large values the only fact that effects RIN value is that the gasoline price is above ethanol (and not how much it is above ethanol), in low values, the price difference matters.

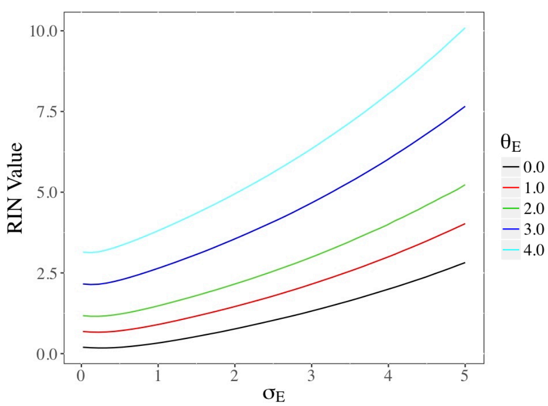
A similar behavior is observed in the GMR scenario. The relation is negative for low levels of volatility and then becomes positive when the volatility is above a certain threshold. Regardless of long-run equilibrium gasoline price, RIN values tend to converge for high levels of gasoline volatility. Though the speed of convergence is considerably slower as compared to the case of GBM. RIN values also diverge faster for higher long-run equilibrium price levels with the increase of $\sigma_E$. This may be attributed to the possibility of early exercise.



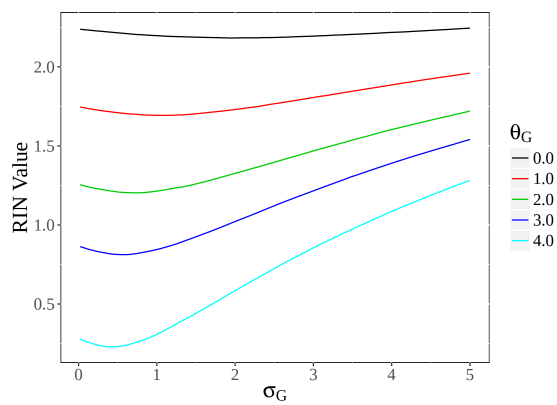

(a) The effect of $\sigma_E$ for GBM




(b) The effect of $\sigma_G$ for GBM




(c) The effect of $\sigma_E$ for GMR



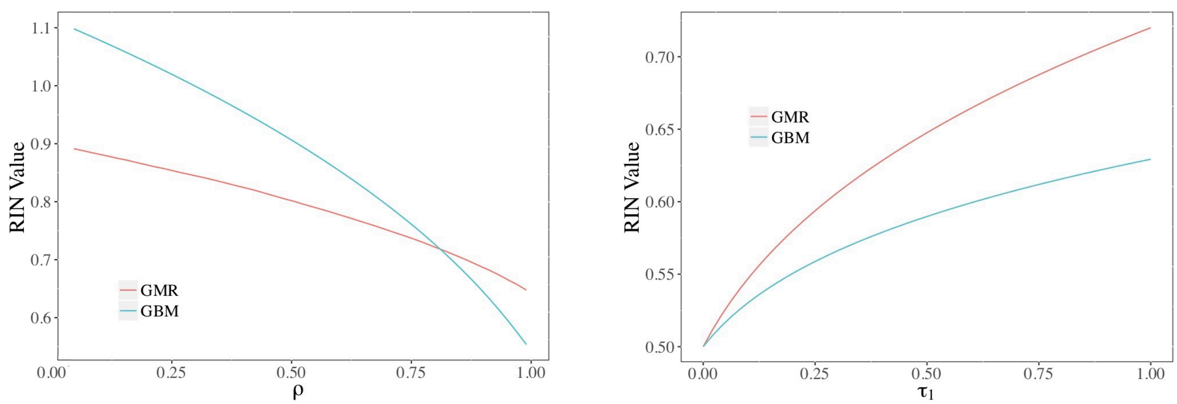

(d) The effect of $\sigma_G$ for GMR

Figure 2: The effect of volatility for different initial prices of ethanol and gasoline. The remaining parameters are as reported in Table 3.

*5.1.3. Correlation and Time to Compliance*

Figure 3 compares the sensitivity of the RIN value to the correlation between the two underlying processes and time to expiry under the two assumptions of GBM and GMR. The graphs illustrate that for both models, the value of RIN has a *negative* relationship with the correlation between the spot price of ethanol and gasoline. Intuitively, this happens because a high correlation between two processes reduces the probability of a "wide gap" between the two and therefore the value of a contract resembling a spread-option.

As one expects, the value of RIN decreases as the compliance date approaches. This result is again intuitive because a shorter time to the maturity reduces the chance of large divergence between the two processes. We also see that the value drops more rapidly in the case of GMR. The possibility of early exercise under GMR is the major reason for this difference.




(a) Correlation (b) Time to expiry

Figure 3: Sensitivities of RIN to correlation and time to expiry under two different underlying behaviors. The initial price values are set as $P_E = 2.5$, $P_G = 2.0$. Other parameters are held constant as reported in Table 3. The effect of the time-to-expiry is stronger for the GMR process because it includes a higher chance of early exercise in addition to the expected pay-off at the maturity day.

*5.1.4. Borrow/Banking Allowance Level*

The effect of the level of banking/borrowing allowance ($\xi$) on the prices is also illustrated in Figure 4. As the formulation in Definition 1 shows, the banking/borrowing allowance

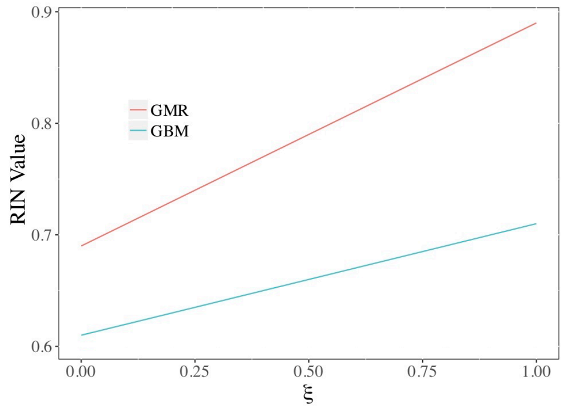


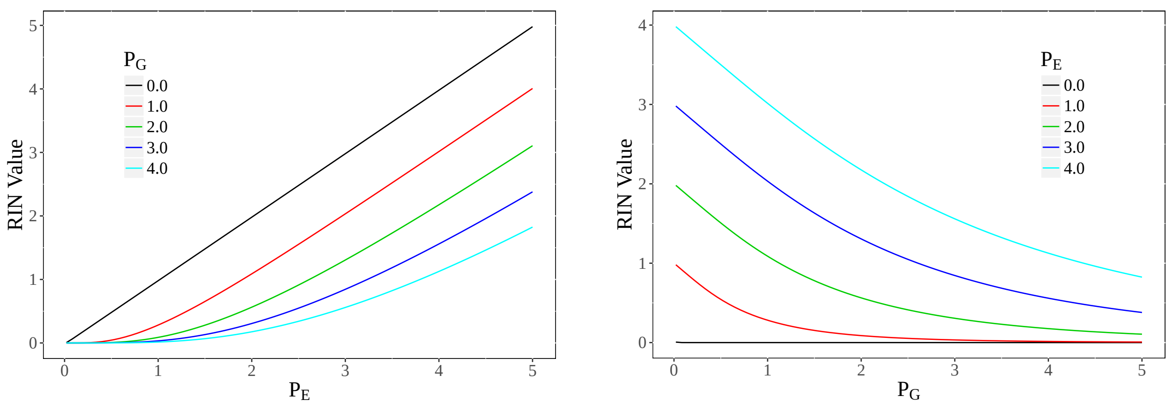

Figure 4: The effect of borrow/banking allowance level on RIN prices. The initial price values are set as $P_E = 2.5$, $P_G = 2.0$. The remaining factors but one are held constant and are as reported in Table 3. The impact of borrowing/banking is stronger for the GBM case because no early optimal exercise takes place under this process.

level has a linear effect on the price of RINs. The intuition behind this effect is that a RIN of this year's vintage, has a $(1 - \xi)$ probability to be exercised before this year's compliance date and a $\xi\%$ chance to be transferred to the next year.

Ceteris paribus, the value of RIN is at its lowest when banking and borrowing is not allowed. As can be inferred from Definition 1, this is because allowing for banking/borrowing adds a second non-negative term to the value of RIN. The increase in the allowance level makes RINs more valuable, as the probability that a RIN would be transferable to the next compliance period increases. The reason being that under both scenarios as the time of expiry is extended, the option value increases.

### *5.1.5. Current Price*

Figure 5 illustrates the effect of the current underlying prices on RIN value in the case of GBM. As it can be seen, *ceteris paribus* the value of RIN decreases for higher levels of initial gasoline price. Though the level of decrease for a unit increase in the price of gasoline gradually decreases until it converges to zero. The reasoning given the setup is evident. As the price of gasoline (and subsequently, the difference between the price of ethanol and gasoline) increases, the chances of ethanol price being above gasoline price by the expiry




(a) The effect of $P_E$

(b) The effect of $P_G$

Figure 5: The effect of initial gasoline and ethanol price on RIN value in the case of GBM. All remaining models' parameters are that of Figure 3. We observe a convex relationship between the current price and RIN's value. The convexity increases as the price of other underlying increases.

date decreases until it is almost surely zero.

### *5.2. Discussions and Interpretations*

Our model provides some new insights for policy makers and firms. We conclude the analysis by discussing a few additional issues.

#### *5.2.1. Early Exercise*

What immediately distinguishes the two underlying scenarios is that unlike GBM, it may be optimal to *exercise* a RIN early when the prices follow GMR. To see the intuition behind this feature, let us first assume the benchmark case of GBM with no convenience yield, where it is *never* optimal to do an early-exercise. For the sake of simplification, first assume there are only two days in a compliance period and banking/borrowing is not allowed. The blender is endowed with one RIN and based on their production projection, they need to have two RINs by the end of the second day. In this situation, the blender always obtains the required RIN by producing ethanol in the first day. In the second day, they use pure gasoline and exercise the RIN, which would be worthless the next day. This holds regardless of the absolute or relative price of ethanol and gasoline in the two days. In a more general framework, when there are $n$ days in the compliance period, $m$ RINs need to be obtained, and production level is the same in all days. The blender obtains RINs by producing ethanol or buying them in the market, but only in the last day do they use the excess RIN. They are indifferent with respect to the time when those RINs are obtained.

On the contrary, in the GMR case the blender may find it optimal to *exercise* their RINs at certain times when the immediate pay-off is sufficiently large. To *exercise* RINs however, one is not necessarily required to own the excess RINs. The blender may in fact choose to use gasoline instead of ethanol during the optimal dates by borrowing the RINs from the future. In other words, the blender obtains the required RINs (by either producing ethanol or buying RINs at market) in those subsequent days when it is not optimal to exercise.

It should also be noted that in the GBM setup, we assumed the convenience yield to be negligible. This allowed us to use the resulting formulation as a benchmark model for valuing RINs. Nevertheless, in reality, convenience yield may have a significant magnitude. In this case, one cannot ignore the possibility of early exercise of RINs and the price dynamics are better captured by the GMR assumptions. In fact, in the presence of convenience yield, price dynamics show a mean-reverting behavior even if the markets are assumed to be very efficient. This behavior is caused by the fact that the mean-reversion property acts similar to the yield but through a very different role.

#### *5.2.2. Number of Years to Borrow and Bank*

Our model identifies the impact of the banking period length (i.e. the maturity date) on the optimal decisions of gasoline producers and highlights trade-offs. On one hand, short-time horizons make it harder for all firms to comply. On the other hand, the main drawback of allowing long time horizons of borrowing is that in the hope of future changes or amendments in the policies, blenders find the incentive to move toward borrowing RINs from the future indefinitely, making the policy effectively useless. Finding the optimal number of years to allow banking/borrowing remains a policy question to be more analyzed.

#### *5.2.3. Bang-bang Exercise*

Exercising RINs is a *bang-bang* type control solution: once it is optimal to exercise, the blender uses pure gasoline, and once it is not, the blender obtains the maximum amount of RIN possible, either through producing ethanol or buying RINs at the market. Currently, there are four distinct ethanol-gasoline blends available in the US: E0 (with no ethanol), E10 (with up to 10% ethanol), E15 (containing up to 15% ethanol) and E85, which is only designated for flexible-fuel vehicles and contains between 51-83% of ethanol (average of 74%). Of these, E10 is by far the two dominant types of fuels in the market, and E85 is the fuel that has been introduced as the more sustainable alternative. Therefore, the only way blenders can produce more than a certain level of RIN would be through using the produced ethanol in E85. This in fact has been one of the incentives behind EPA's efforts in increasing mandate levels (EIA (2016)). Our model assumes blenders are able to perform this switch with no cost.

#### *5.2.4. RINs versus Swing Options*

RINs are comparable to the class of swing options with one exercise right. The value of the contract in this case is equivalent to that of a American (Bermudan) option. But the value of RINs is also different from a swing contract due to certain features. Conventional swing contracts are written on a single underlying commodity; whereas, as mentioned above, RIN value reflects the spread between the price of two commodities. The compliance level discussed above and the possibility of banking and borrowing between years are among other characteristics that distinguish RINs from swing contracts (and for that matter exchange options). In more details, the industry being able to carry net deficit/surplus, combined with the option value associated with RINs may lead to instances where future vintage RINs trade at a premium, as compared to current vintages. As pointed out by Lade et al. (2015), this demonstrates that the market expects the banking constraints to bind. Consequently, if there is no cap on banking/borrowing restrictions, the constraints do not bind, and there is no price difference for same RINs with different vintages. This price wedge can also be easily observed in Definition 1. The price of a generated RIN transferable to next year, includes an additional term compared to a RIN that expires by the end of the current year.

## 6. Conclusion

The United States and EU countries have mandates for the minimum use of renewable fuels. One of these mandates in the US, which is part of the current Renewable Fuel Standards (RFS), is the requirement for gasoline producers (blenders) to use about 10% biofuels (ethanol) in their transportation fuel production. Similar policies have been adapted in different European countries. While these policies have differences in design, execution and flexibility for adaption, they all allow for the existence of a market for trading renewable fuel certificates such as Renewable Identification Numbers. A core feature common in the design of these markets is that they give an option value to the certificate being traded. In this paper, we focused on RINs in the US market to examine the value of these certificates, their relation with parameters that explain the price of gasoline and ethanol, and their effect on production.

This paper models the price of RINs as an American spread option. We used two different models for the price of ethanol and gasoline as the underlying processes of this option, namely Geometric Brownian motion, and Geometric mean-reverting process. In this *real options* framework, the value of RIN reflects the supremum of the expected difference between the price of ethanol and gasoline. As the option is American, it is possible for the owner to find early exercising the option (i.e., RIN) optimal. In this setup, RIN is exercised when the blender decides to use gasoline instead of the required number of RINs in that day of production. Using RINs is a bang-bang type control. That is, the control only takes values from a finite set. In more details, at times when it is optimal to exercise the RIN, the blender decides to blend pure gasoline, and use their previously acquired RINs instead of ethanol, buy RINs at the market, or borrow the RINs from future. When it is not optimal to exercise RIN, the blender decides to obtain the maximum amount of RIN possible, either through production (and blend of ethanol) or buying RINs in the market.

The GBM setup leads to a closed-form solution for the price of RINs. Under this setup, the blender never finds it optimal to use RINs instead of ethanol. Though the commodity market literature does not favor GBM setup for explaining the price of commodities (for various reasons the most important of which are mentioned in the text), the resulting closed-form solution allows for simple and rapid computation of prices.

Next, we consider the prices to follow the more advocated dynamic of Gemoetric mean-reverting processes and model the value of RIN. Under this assumption, the value of RIN can be obtained with a lower and an upper bound. We also extend the GBM setup for the underlying and introduce the GBM dynamics to Merton's jump. The resulting model leads to an approximation of the price of RINs. With the derived valuations, we perform a set of numerical simulation to examine the sensitivity of RIN prices to various underlying parameters. In particular, we study the effect of the correlation between gasoline and ethanol prices, time left until the compliance date, banking/borrowing allowance level, initial price of gasoline and ethanol (under GBM setup), long-run equilibrium price levels (under GMR setup), and volatility.

Results illustrate a negative relation between correlation and price of RIN and a more rapid decline of RIN value as the compliance date approaches in the GMR scenario. In accordance with definition and analytic formulation, borrow/banking allowance level has a linear effect on prices. When analyzing the role of volatility, we observe a U-shape relationship between the volatility of both underlyings and RIN price under both scenarios. In addition, graphs show that at high volatility levels RIN prices converge.

The results of this paper can be used to examine renewable certificate markets from various aspects. The framework setup and the resulting valuation models can be used in designing similar markets in other countries or in amending the current markets. This is particularly useful when the blenders are failing to comply with the policy mandates or the set targets have proven to be too ambitious. An example of these instances is in December 2015 when the EPA decided to lower the mandate requirement, which was an acknowledgment of the difficulty of compliance with the original goals. The results can also be used to study the affect of mandates on production plan of blenders.

The framework used in this study can be applied to examine floor-and-trade mechanisms in other markets. In the RINs market in particular, this work can be further extended to better capture the market dynamics. The price of fuel, and especially ethanol is different in different regions of the US. A possible future step of this work may be incorporating a framework similar to Litzenberger and Rabinowitz (1995) to differentiate between producers. As previously explained, there are a number of fundamental questions with regards to policy design in markets similar to RINs that still need to be better addressed. These include blended biofuel share of the fuel, optimal level of allowance for borrowing and banking, optimal size of compliance period, penalty per shortage, and subsidies. For the purpose of simplification, we have not taken into account nested mandates. Future studies may incorporate nested mandates, which can be useful especially in designing and amending the policies and its affects on the trade market.

Although latent in our valuation model, due to space limitations of the current paper we do not explicitly present the optimal production problem of the representative producer. Future research can be more explicit regarding the optimal time-varying blending rates over time. A follow-up study can characterize the optimal threshold for generating excess RINs and also the optimal multi-unit exercise strategy for an inventory or RINs in the presence of "flow limitations" (i.e. low and upper bounds on the blending rate).

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## Appendix A. Biofuels Markets and Renewable Identification Numbers (RINs)

The Environmental Protection Agency (EPA) in the US imposes Renewable Fuel Standards (RFS) using biofuel blending mandates. The mandates, which are set on an annual basis, should be met by final motor gasoline producers and importers (hereinafter referred to as blenders). Meeting this mandate compliance is verified by the blender submitting a sufficient quantity of Renewable Identification Numbers (RINs) to EPA. The important feature is that RINs, generated through blending biofuels, can also be bought and sold in the market.

The Energy Policy Act of 2005 established Renewable Fuel Standards (RFS) requiring the use of biofuels (such as ethanol) in the automative fuel supply in the US. The RFS were later amended and broadened under the Energy Independence and Security Act of 2007, and are referred to as RFS2. The RFS2 requires blenders to meet four Renewable Volume Requirements (RVOs) that are announced based on the ratio percentage of renewable to non-renewable fuel production. Blenders prove meeting their share of mandates by using a tracking system known as renewable identification numbers (RINs). A RIN generates as a 38-character code upon the use of one gallon of renewable fuel produced or imported, and is designed to make meeting the required mandates more flexible. After the renewable fuel is blended, the blender can detach the RIN and either store it for compliance or sell it at the market as a commodity. The possibility of trade gives some blenders an incentive to blend biofuels beyond their share of mandate, and encourages some to meet the mandate by obtaining RINs rather than directly blending biofuels in the product. To lower the cost of compliance, the Environmental Protection Agency (EPA) also allows RINS to be banked and borrowed up to 20% of the projected production of the next year. However, in 2013, following concerns over high compliance costs and lower than expectation investment in advanced biofuel, the EPA was led to reduce the RVOs for the next three consecutive years.

Similar policies have also been adopted in Europe. The 2009 EU Renewable Energy

Directive sets a 10% minimum target for renewable energy consumption in the transportation sector for the member states by 2020. In addition, the Fuel Quality Directive requires the road transport fuel mix in the EU to be 6% less carbon intensive than a fossil diesel and gasoline baseline by 2020. Since, the countries have been pursuing these goals mainly by setting minimum biofuel (bio-ethanol and bio-diesel) use mandate levels. To allow for adaption, the 28 countries use target increase policies of the minimum mandate level on a yearly basis. These targets are different across different countries. For example, though some such as Finland have already surpassed the 10% and are undertaking more ambitious goals, other members such as Greece are still far below the target level. A detailed description of the current policies by the EU countries is provided by Lieberz (2017). Similar to the US, the availability of a market mechanism known as certificate trading has offered flexibility to smoothing the transition.

## Appendix B. Proof of Proposition 1

Assuming no banking constraint, the value of RIN under the risk-neutral measure $\mathbb{Q}$ (which will be derived later) is

$$p = e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(P_E(T) - P_G(T)), 0] \tag{B.1}$$

*Corollary A.1.* Assuming $dB_E(t)dB_G(t) = \rho dt$, processes $W_1(t)$ and $W_2(t)$, defined as follows are independent Brownian motions.

$$\begin{aligned} B_E(t) &= W_1(t) \\ B_G(t) &= \int_0^t \rho dW_1(s) + \int_0^t \sqrt{1-\rho^2} dW_2(s) \end{aligned} \tag{B.2}$$

*proof.* First note that $dW_1(t) = dB_E(t)$ and:

$$dW_2(t) = \frac{-\rho}{\sqrt{1-\rho^2}} dB_E(t) + \frac{1}{\sqrt{1-\rho^2}} dB_G(t) \tag{B.3}$$

Which leads to:

$$W_2(t) = \int_0^t \frac{-\rho}{\sqrt{1-\rho^2}} dB_E(u) + \int_0^t \frac{1}{\sqrt{1-\rho^2}} dB_G(u) \tag{B.4}$$

$W_1(t)$ is obviously a Brownian motion (BM). Also $W_2(0) = 0$. From equation B.4 and the fact that $B_E$ and $B_G$ are both BM and thus martingales (i.e., the stochastic integral is also a martingale), we conclude $\mathbb{E}[W_2(t)] = 0$. Thus $W_2(t)$ is a continuous martingale process. The following also holds:

$$\begin{aligned} &dW_2(t).dW_2(t) = \\ &\quad \frac{\rho^2}{1-\rho^2} dB_E dB_E - \frac{\rho}{1-\rho^2} dB_E dB_G - \frac{\rho}{1-\rho^2} dB_G dB_E + \frac{1}{1-\rho^2} dB_G dB_G \\ &\quad = \frac{\rho^2}{1-\rho^2} dt - \frac{\rho}{1-\rho^2} \rho dt - \frac{\rho}{1-\rho^2} \rho dt + \frac{1}{1-\rho^2} dt = dt \end{aligned} \tag{B.5}$$

Meaning $[W_2, W_2]_{(t)} = t$. Using Levy theorem, one can conclude $W_2(t)$ is also a BM. It remains to show $W_1$ and $W_2$ are independent.

$$\begin{aligned} dW_1(t)dW_2(t) &= dB_E(t)\left(\frac{-\rho}{\sqrt{1-\rho^2}} dB_E(t) + \frac{1}{\sqrt{1-\rho^2}} dB_G(t)\right) \\ &= -\frac{\rho}{1-\rho^2} dt + \frac{\rho}{1-\rho^2} dt = 0 \end{aligned} \tag{B.6}$$

Again, using Levy theorem, it is concluded that $W_1$ and $W_2$ are independent Brownian motions.

Using corollary A.1, one can describe the price dynamics as:

$$\frac{dP_G(t)}{P_G(t)} = \mu_E dt + \sigma_E dW_1(t)$$
$$\frac{dP_E(t)}{P_E(t)} = \mu_G dt + \sigma_G(\rho dW_1(t) + \sqrt{1-\rho^2} dW_2(t))$$

where $W(t) = (W_1(t), W_2(t))$ is a two-dimensional Brownian motion.

*Corollary A.2.* There is no arbitrage in the given dynamics and a risk-neutral measure exists.

*proof.* There are 2 equations and 2 unknowns for defining the market price of risks processes $\Theta_1, \Theta_2$. According to Shreve (2004), these equations are:

$$\mu_i - r = \sum_{j=1}^{2} \sigma_{ij} \Theta_j \tag{B.7}$$

where $\sigma_{11} = \sigma_E, \sigma_{12} = 0, \sigma_{21} = \rho\sigma_G, \sigma_{22} = \sqrt{1-\rho^2}\sigma_G$ This leads to:

$$\begin{aligned} \Theta_1 &= \frac{\mu_E - r}{\sigma_E} \\ \Theta_2 &= \frac{(\mu_G - r)\sigma_E - (\mu_E - r)\rho\sigma_G}{\sigma_E \sigma_G \sqrt{1-\rho^2}} \end{aligned} \tag{B.8}$$

Also, it is easy to verify that:

$$\mathbb{E}\Big( \exp(\frac{1}{2} \int_0^T (\Theta_1^2 + \Theta_2^2) dt) \Big) < \infty$$

Therefore, based on the first fundamental theorem of asset pricing, no arbitrage exists in the market and a risk-neutral measure (equivalent to the original measure) exists, which will be subsequently derived.

*Corollary A.3.* The risk neutral dynamics when the solution to market price of risk follows equation B.8 is given by:

$$\begin{aligned} dP_G(t) &= rP_G(t)dt + \sigma_E d\tilde{B_E}(t) \\ dP_E(t) &= rP_E(t)dt + \sigma_G d\tilde{B_G}(t) \end{aligned} \tag{B.9}$$

where $\tilde{B}_i = B_i + \int_0^t \gamma_i(u)du$ and $\gamma_i(t) = \sum_{j=1}^2 \frac{\sigma_{ij}}{\sigma_i}\Theta_j$. Further $d\tilde{B}_E d\tilde{B}_G = \rho dt$

*proof.* $Z$ is a Radon-Nykodym derivate such that:

$$Z(t) = \exp\{-\int_0^t \Theta(u).dW(u) - \frac{1}{2}\int_0^t ||\Theta(u)||^2 du\} \tag{B.10}$$

Setting $Z = Z(T)$, then $\mathbb{E}Z = 1$, and under this probability measure $\mathbb{Q}$ given by

$$\mathbb{Q}(A) = \int_A Z(\omega)d\mathbb{P}(\omega)$$

it can be stated that

$$\tilde{W}_j(t) = W_j(t) + \int_0^t \Theta_j du$$

are independent Brownian motions, according to Girsanov's theorem. Note that $\mathbb{P}$ is the original probability measure. First it is shown that $\tilde{B}_i$ is a Brownian motion under $\mathbb{Q}$. Note that by definition of $B_i(t)$:

$$\begin{aligned} \tilde{B}_i(t) &= \sum_{j=1}^2 \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i} dW_j(u) + \gamma_i(u)\Big) \\ &= \sum_{j=1}^2 \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i}(dW_j(u) + \Theta_i(u)du)\Big) \\ &= \sum_{j=1}^2 \int_0^t \Big(\frac{\sigma_{ij}}{\sigma_i} d\tilde{W}_j(t)\Big) \end{aligned} \tag{B.11}$$

Using Levy Theorem, one can show $\tilde{B}_i(t)$ is a Brownian motion under the original probability measure.[$\tilde{B}_i(t) = 0$. Also $\mathbb{E}[\tilde{B}_i] = 0$, thus $\tilde{B}_i(t)$ is a continuous martingale and $d\tilde{B}_E(t)d\tilde{B}_G(t) = dt$] By theorem 4.3.1 each of the Ito integrals in the definition of $\tilde{B}_i(t)$ is a continuous martingale (starting at zero). The quadratic variation is

$$d\tilde{B}_i.d\tilde{B}_i = \sum_{j=1}^{2} \frac{\sigma_{ij}^2}{\sigma_i^2} dt = dt$$

. Therefore, again by using Levy Theorem, $\tilde{B}_i(t)$ is a martingale under $\mathbb{Q}$. Given the dynamics in the question and the relation between $B_i$ and $\tilde{B}_i$ we can show:

$$\begin{aligned}
\frac{dP_i}{P_i} &= \mu_i dt + \sigma_i dB_t \\
&= \mu_i dt + \sigma_i \sum_{j=1}^{2} \frac{\sigma_{ij}}{\sigma_i} dW_j(t) \\
&= \mu_i dt + \sigma_i \sum_{j=1}^{2} \frac{\sigma_{ij}}{\sigma_i} (d\tilde{W}_j(t) - \Theta_j dt) \\
&= \mu_i dt + \sigma_i (d\tilde{B}_i(t) - \sum_{j=1}^{2} \frac{\sigma_{ij}\Theta_j}{\sigma_i} dt) \\
&= (\mu_i - \sum_{j=1}^{2} \sigma_{ij}\Theta_j) dt + \sigma_i d\tilde{B}_i(t) \\
&= r dt + \sigma_i d\tilde{B}_i(t)
\end{aligned} \tag{B.12}$$

Now, to show the correlation between the new Brownian motions:

$$\begin{aligned}
d\tilde{B}_E d\tilde{B}_G &= (dB_E(t) + \gamma_1(t)dt)(dB_G(t) + \gamma_2(t)dt) \\
&= dB_E(t)dB_G(t) = \rho dt
\end{aligned} \tag{B.13}$$

*Corollary A.4.* Risk-neutral measure $\mathbb{Q}$ is unique.

*proof.* As the driving Brownian motions are the only source of uncertainty, the only way multiple risk-neutral measures can arise is via multiple solutions to the market price of risk equations. Equation B.8 reveals the market prices of risk $\Theta_1, \Theta_2$, have unique solutions. It is concluded that the risk-neutral measure is also unique.

The value of RIN under the risk-neutral measure is

$$\begin{aligned} p &= e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(P_E(T) - P_G(T)), 0] \\ &= e^{-rT}\mathbb{E}_{\mathbb{Q}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0)P_G(T)] \end{aligned} \tag{B.14}$$

or in other words, the price of $P_E$ can be explained in terms of $P_G$ and still remain a GBM. This is the basic idea behind solving the question when the strike price $K$ is equal zero. Further, the risk-neutral dynamics can be explained by:

$$\begin{aligned} \frac{dP_G}{P_G} &= rdt + \sigma_E d\tilde{W}_1(t) \\ \frac{dP_G}{P_G} &= rdt + \sigma_G(\rho d\tilde{W}_1(t) + \sqrt{1-\rho^2}d\tilde{W}_2(t)) \end{aligned} \tag{B.15}$$

Where $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ are independent Brownian motions defined in corollary A.3. Proof is provided in corollary A.1. Note that by applying Ito's lemma:

$$\begin{aligned} \frac{d(P_E(t)/P_G(t))}{P_E(t)/P_G(t)} =& rdt + \sigma_G[\rho d\tilde{W}_1(t) + \sqrt{1-\rho^2}d\tilde{W}_2(t)] - [rdt + \sigma_E d\tilde{W}_1(t)] \\ &+ \sigma_E^2 dt + [rdt + \sigma_E d\tilde{W}_1(t)][rdt + \sigma_G \rho d\tilde{W}_1(t) + \sqrt{1-\rho^2}d\tilde{W}_2(t)] \\ =& (\rho\sigma_G - \sigma_E)d\tilde{W}_1(t) + \sqrt{1-\rho^2}\sigma_G d\tilde{W}_2(t) + \sigma_E(\sigma_E - \rho\sigma_G)dt \end{aligned} \tag{B.16}$$

Now, defining a new probability measure, $\mathbb{C}$, with Radon-Nikodym derivative can be done using Girsanov theorem:

$$\frac{d\mathbb{C}}{d\mathbb{Q}} = \frac{1}{P_G(0)}P_G(T) = \exp(-\frac{1}{2}\sigma_E^2 T + \sigma_E dW_1(T)) \tag{B.17}$$

Under this new measure, according to Grisanov theorem, $\hat{W}_1(t) = \tilde{W}_1(t) - \sigma_E t$ is a Brownian motion (so is $\hat{W}_2(t) = \tilde{W}_2(t)$). Therefore:

$$\begin{aligned} p =& e^{-rT}\mathbb{E}_{\mathbb{P}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0)P_G(T) \\ &* \exp(\frac{1}{2}\sigma_E^2(T) - \sigma_E W_1(T) + (r - \frac{1}{2}\sigma_E^2)T + \sigma_E W_1(T))] \\ =& P_G(0)\mathbb{E}_{\mathbb{P}}[\max(\frac{P_E(T)}{P_G(T)} - 1, 0)] \end{aligned} \tag{B.18}$$

Therefore, using the result above, one can describe $P_E(t)/P_G(t)$ under $\mathbb{C}$ as a GBM with volatility $\sigma$ as:

$$\frac{d(P_E(t)/P_G(t))}{P_E(t)/P_G(t)} = \sigma dW_3(t) \tag{B.19}$$

In the above equation,

$$\begin{cases} \sigma^2 = \sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G \\ dW_3(t) = \frac{\rho\sigma_G - \sigma_E}{\sigma} d\hat{W}_1(t) + \frac{\sqrt{1-\rho^2}\sigma_G}{\sigma} d\hat{W}_2(t) \end{cases} \tag{B.20}$$

where the volatility $\sigma$ equality holds because:

$$[(\rho\sigma_G - \sigma_E)d\hat{W}_1(t) + (\sqrt{1-\rho^2}\sigma_G)d\hat{W}_2(t)]^2 = (\sigma_E^2 + \sigma_G^2 - 2\rho\sigma_E\sigma_G)dt$$

Also, $W_3(t)$ is a BM under $\mathbb{C}$ because i) $\hat{W}_1(t)$ and $\hat{W}_2(t)$ are BMs, independent and thus martingales. Subsequently, their stochastic integral is also a martingale. ii) $\mathbb{E}W_3(0) = 0$. Therefore, $W_3(t)$ is a continuous martingale. iii)

$$\begin{aligned} dW_3(t).dW_3(t) &= [\frac{(\rho\sigma_G - \sigma_E)}{\sigma} d\hat{W}_1(t) + \frac{(\sqrt{1-\rho^2}\sigma_G)}{\sigma} d\hat{W}_2(t)]^2 \\ &= (\frac{(\rho^2\sigma_G^2 + \sigma_E^2 - 2\rho\sigma_E\sigma_G)}{\sigma^2} + \frac{(1-\rho^2)\sigma_G^2}{\sigma^2}) = dt \end{aligned} \tag{B.21}$$

From here on, the solution is relatively straightforward. Using the BSM formula for a

European Call Option, the derivative security can be priced as:

$$p = P_E(0)N(d_1) - P_G(0)N(d_2) \tag{B.22}$$

where

$$d_1 = \frac{\ln(P_E(0)/P_G(0))}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \quad d_2 = \frac{\ln(P_E(0)/P_G(0))}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}$$

and $\sigma$ is defined as in equation B.20. This is equivalent to the well-known Margrabe (1978) equation.

## Appendix C. Deriving risk-neutral dynamics of mean-reverting processes

Suppose real world dynamics of the prices follow

$$\begin{aligned} P_i(t) &= \exp(\tilde{\theta}_i + X_i(t)) \\ dX_i(t) &= -\tilde{\alpha}_i X_i(t)dt + \sigma_i d\tilde{B}_i(t) \end{aligned} \tag{C.1}$$

where $i \in \{E, G\}$, $\tilde{\alpha}_i$s are the constant mean reversion coefficients, estimated using regressing historical data. $\tilde{B_E}(t)$ and $\tilde{B_G}(t)$ are standard Brownian motions with correlation $\rho$ under real world measure $\mathbb{P}$. $\exp(\tilde{\theta}_i)$ is known as the asymptotic mean reversion level and $\sigma_i$ is the volatility.

To avoid redundancy, some steps that are similar to the proof in Appendix B are omitted. In order to provide the risk-neutral measure required for valuation, we use the market price of risk. The market prices of risk $\lambda_i(t)$ associated with the Brownian motions, $\tilde{B_E}(t)$ and $\tilde{B_G}(t)$, are assumed to be affine in $X_i(t)$, more specifically:

$$\lambda_i(t) = \frac{a_i + b_i X_i(t)}{\sigma_i} \tag{C.2}$$

with $a_i$ and $b_i$ as constants. Using Girsanov's theorem one can show that under the equivalent risk-neutral measure $\mathbb{Q}$, the drift adjusted processes, $B_i(t) = \int_0^t \lambda_i(s)ds + \tilde{B}_i(t)$ are also standard Brownian motions with correlation $\rho$. Consequently, the price dynamics under this new measure are:

$$\begin{aligned} P_i(t) &= \exp\left(\theta_i + X_i(t)\right) \\ dX_i(t) &= -\alpha_i X_i(t)dt + \sigma_i dB_i(t) \end{aligned} \tag{C.3}$$

where $\alpha_i = \tilde{\alpha}_i - b_i$, and $\theta_i = \tilde{\theta}_i + a_i/\alpha_i$.

## Appendix D. Proof of Proposition 2

This proof is similar to the proof of Longstaff and Schwartz (2001) convergence proposition. By definition, the underlying processes are $\mathscr{F}_t$ measurable. Similarly, the immediate exercise value is also $\mathscr{F}_t$ measurable. $C_{t_m}$, which is a linear function of $\mathscr{F}_t$ measurable functions of the underlyings is also $\mathscr{F}_t$ measurable. Thus the event of exercise value being greater than or equal to $C_{t_m}$ is in $\mathscr{F}_t$. The LSMC method is thus a stopping time problem.

If the present value of the stopping rule is shown by $E_\theta$, $V_t \geq E_\theta$ as $V_t$ is by definition the supremum of the present values over the set of all stopping times. As $C_{t_m}$ is the same across all paths, the discounted cash flows $LSMC(x, y, d, m)$ are i.i.d. The Strong Law of Large Numbers implies:

$$\Pr[\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} LSMC(x, y, d, m) = E_\theta] = 1 \tag{D.1}$$

Combined with inequality $V_t \geq E_\theta$ and using Proposition 1 proof can be obtained.

## Appendix E. Proof of Proposition 3

Equation 4 provides a closed form valuation of RIN under GBM assumptions. Using the notations we have

$$\lim_{\sigma\to\infty} d_{1\pm} = \pm\infty \quad \lim_{\sigma\to\infty} d_{2\pm} = \pm\infty \tag{E.1}$$

Subsequently, when $R_t \leq \bar{R}_i$:

$$\lim_{\sigma \to \infty} V_t = (1-\xi)P_E + \xi P_E = P_E \tag{E.2}$$

And similarly $\lim_{\sigma \to \infty} V_t = \frac{\bar{R}_i}{R_t} P_E$ when $R_t > \bar{R}_i$

## Appendix F. Extension to Jump-Diffusion Models

The inclusion of jumps in price dynamics is particularly useful in modeling commodity prices and has been studied and advocated extensively in the previous literature (e.g., Hilliard and Reis (1998), Deng (2000), Geman (2009)). This section examines this application when gasoline and ethanol price dynamics are introduced to Merton's jump.

### *Appendix F.1. GBM dynamics*

Proposition 1 provides a simple and useful closed-form valuation of RIN under GBM setup. This valuation can be adjusted to approximate the price of RINs when one allows for jumps in the risk-neutral dynamics of the underlying prices. Assume the neutral-risk price dynamics follow:

$$\frac{dP_i(t)}{P_i(t-)} = (r - \lambda_i \mu_i)dt + \sigma_i dB_i t(t) + (e^{J_i(t)} - 1)dN_i(t) \tag{F.1}$$

where again $i \in \{E, G\}$. $J_i(k)_{k \geq 1}$ are two independent sequences of Gaussian random variables with distribution $N(m_i, s_i^2)$ and $\mu_i = e^{m_i + s_i^2/2} - 1$. $\mathbb{E}\{dB_E(t)dB_G(t)\} = \rho dt$. $N_E$ and $N_G$ are two Poisson processes with intensities $\lambda_E$ and $\lambda_G$, respectively, that are independent from each other and from $B_E$ and $B_G$. A Merton jump process is interpreted as follows: at the time the Poisson process $N_i(t)$ jumps for the $k-$th time, the price $P_i(t)$ jumps by an amount of $P_i(t-)(e^{J_i(k)} - 1)$. The spread option pricing shown and proved by Carmona and Durrleman (2003) can be used to approximate the value of RINs in this setup.

**Proposition 4.** *The price of RIN when the underlying prices follow equation F.1 can be approximated by:*

$$\begin{aligned}\hat{V}_t = (1-\xi)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-(\lambda_E+\lambda_G)T_1}\frac{(\lambda_E T_1)^j(\lambda_G T_1)^i}{i!j!}P_1 \\ +\xi\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} e^{-(\lambda_E+\lambda_G)T_2}\frac{(\lambda_E T_2)^j(\lambda_G T_2)^i}{i!j!}P_2\end{aligned} \tag{F.2}$$

*When $R_t \le \bar{R}_i$, and $\frac{\bar{R}_i}{R_t}\hat{V}_t$ when $R_t > \bar{R}_i$ In this equation $P_1$ and $P_2$ are derived by a slight adjustment of the components in the RIN value under GBM setup. For $\iota \in \{1, 2\}$*

$$P_\iota = x_{E,\iota}\Phi(d_{\iota,+}) - x_{G,\iota}\Phi(d_{\iota,-})$$

*where*

$$\begin{aligned}
x_{E,\iota} &= P_E(t)\exp\left(-\lambda_E\mu_E T_\iota + j(m_E + s_E^2/2)\right)\\
x_{G,\iota} &= P_G(t)\exp\left(-\lambda_G\mu_G T_\iota + i(m_G + s_G^2/2)\right)\\
d_{\iota,\pm} &= \frac{\ln(x_{E,\iota}/x_{G,\iota})}{\bar{\sigma}_\iota\sqrt{T_\iota - t}} \pm \frac{\bar{\sigma}_\iota}{T_\iota - t}\\
\bar{\sigma}_\iota{}^2 &= \bar{\sigma}_{E,\iota}^2 + \bar{\sigma}_{G,\iota}^2 - 2\bar{\rho}_\iota\bar{\sigma}_{E,\iota}\bar{\sigma}_{G,\iota}\\
\bar{\sigma}_{E,\iota} &= \sqrt{\sigma_E^2 + js_E^2/T_\iota}\\
\bar{\sigma}_{G,\iota} &= \sqrt{\sigma_G^2 + is_G^2/T_\iota}\\
\bar{\rho}_\iota &= \frac{\rho\sigma_E\sigma_G}{\bar{\sigma}_{E,\iota}\bar{\sigma}_{G,\iota}}
\end{aligned}$$

*proof:.* See Carmona and Durrleman (2003).

The formula consists of the summation of infinite series, and this is the main source of approximation. Choosing what number of terms to include is based on the prescribed error threshold.

*Appendix F.2. GMR dynamics*

Let us modify the risk-neutral dynamics of 7 such that the risk-neutral price dynamics follow:

$$\begin{aligned} P_i(t) &= \exp\left(\theta_i + X_i(t)\right) \\ dX_i(t) &= -\alpha_i X_i(t-)dt + \sigma_i dB_i(t) + (e^{J_i(t)} - 1)X_i(t-)dN_i(t) \end{aligned} \tag{F.3}$$

such that the variables are defined similar to equation F.1. Using the Ito formula for finite activity jump processes, again setting $x = \ln P_E - \theta_E$ and $y = \ln P_G - \theta_G$, and taking expectation under the risk-neutral measure yields to the following the following parabolic integro-differential equation (PIDE):

$$\begin{aligned} &\frac{\partial V}{\partial t} + \alpha_E(\theta_E - x)\frac{\partial V}{\partial x} + \alpha_G(\theta_G - y)\frac{\partial V}{\partial y} \\ &+ \frac{1}{2}\left(\sigma_E^2 \frac{\partial^2 V}{\partial x^2} + 2\rho\sigma_E\sigma_G \frac{\partial^2 V}{\partial x \partial y} + \sigma_G^2 \frac{\partial^2 V}{\partial y^2}\right) \\ &+ \lambda_E \int_R [V(xe^{J_E}, y, t) - V(x, y, t)] + \lambda_G \int_R [V(x, ye^{J_G}, t) - V(x, y, t)] - rV = 0 \end{aligned} \tag{F.4}$$

with boundary conditions, value matching and smooth pasting conditions identical to that of equation 8. This PIDE cannot be solved explicitly, but one can use different numerical techniques to estimate the value of RIN under this assumption.